Journal of Engineering Physics and Thermophysics, Vol. 75, No. 1, 2002

MODELING OF ROTATING AND RECIRCULATION FLOWS ON THE BASIS OF A HYBRID TWO-PARAMETER *k*–ε MODEL

V. A. Fafurin

UDC 532.542.4

A modified two-parameter model of turbulence for calculation of flows with a considerable flow curvature is considered. Account for the effects of anisotropy is based on correction of empirical coefficients, which enter into the equation of transfer of dissipation of turbulent kinetic energy, using the Richardson number. It is suggested to apply a correction not to the entire region of calculation, but to the portions where the Richardson number takes on certain values. Satisfactory agreement of calculation results with experimental data of different authors and a slight increase in computational costs allow one to draw the conclusion on the possibility of employing the suggested approaches in engineering practice.

Introduction. One of the unresolved problems of modern physical science is the problem of turbulence. Different means of prediction of turbulent flows are developed at present on the basis of both direct modeling and solution of the Reynolds equations closed by any model. Proceeding from the now available numerical algorithms and computational means, the second approach is the most acceptable for engineering practice [1].

Two-parameter models of turbulence and relations for Reynolds stresses can be used for such classes of flows as rotating or recirculation flows [2, 3]. As compared to the two-parameter model, relations for Reynolds stresses are able to improve, to a certain degree, prediction of rotating or recirculation flows; however, their use leads to a substantial increase in computational difficulties [4–6]. At the same time, an engineering technique must combine an adequate mathematical model and an economical method of calculation by means of this model. Thus, the advantages of the models for Reynolds stresses are not so obvious as might appear at first sight.

One of the main drawbacks of a standard two-parameter *k*–ε model in calculation of rotating and recirculation flows is the primordial prerequisite for the isotropic character of turbulence. However, it is known that a considerable flow curvature observed in recirculation zones leads to a substantial manifestation of anisotropy and, as a result, to poorer calculation results. Use of the Richardson number is one of the means for taking into account turbulence anisotropy. The Richardson number is introduced into the expression for the coefficient C_2. In so doing, a dissipative term, which enters into the equation of transfer for the rate of dissipation of turbulent kinetic energy, is corrected [7].

However, the use of a modification of the constant C_2 for the entire flow region seems to be not quite expedient. The coefficient C_2 for a standard version of the *k*–ε model is chosen from the condition of best coincidence of calculated and experimental data for flows with streamlines without a curvature. At the same time, a flow with zones of recirculation twist also has zones with a slight curvature of flow, and the use of the modification in these zones will probably lead to poorer calculation results as compared to the standard version of the model.

Kazan State Technological University; email: fva@kstu.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 75, No. 1, pp. 76–81, January–February, 2002. Original article submitted February 21, 2000; revision submitted March 27, 2001.

TABLE 1. Quantities Entering into Eq. (1) for the Case of the k–ε Model

Φ	Γ_Φ	S_Φ
1	0	0
u	ν_{eff}	$\frac{1}{r}\left\{\frac{\partial}{\partial x}\left(\nu_{\mathrm{eff}}\, r\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial r}\left(\nu_{\mathrm{eff}}\, r\frac{\partial v}{\partial x}\right)\right\}-\frac{\partial}{\partial x}\left(p+\frac{2}{3}k\right)$
v	ν_{eff}	$\frac{1}{r}\left\{\frac{\partial}{\partial x}\left(\nu_{\mathrm{eff}}\, r\frac{\partial u}{\partial r}\right)+\frac{\partial}{\partial r}\left(\nu_{\mathrm{eff}}\, r\frac{\partial v}{\partial r}\right)\right\}-\frac{\partial}{\partial r}\left(p+\frac{2}{3}k\right)-2\nu_{\mathrm{eff}}\frac{v}{r^2}$
w	ν_{eff}	$-\nu_{\mathrm{eff}}\frac{w}{r^2}-\frac{vw}{r}-\frac{w}{r}\frac{\partial\nu_{\mathrm{eff}}}{\partial r}$
k	$\frac{1}{\mathrm{Re}}+\frac{\nu_\tau}{\sigma_k}$	$P-\varepsilon$
ε	$\frac{1}{\mathrm{Re}}+\frac{\nu_\tau}{\sigma_\varepsilon}$	$\frac{\varepsilon}{k}(C_1P-C_2\varepsilon)$

$$\nu_{\mathrm{eff}}=\frac{1}{\mathrm{Re}}+\nu_\tau$$

$$\nu_\tau=C_\mu\frac{k^2}{\varepsilon}$$

$$P=\nu_\tau\left\{2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial r}\right)^2+2\left(\frac{v}{r}\right)^2+\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial x}\right)^2+\frac{\partial u}{\partial x}+\left[r\frac{\partial}{\partial r}\left(\frac{w}{r}\right)\right]^2\right\}$$

Note. To obtain the system of equations, it is necessary to successively substitute into Eq. (1) the expressions for Φ, Γ_Φ, and S_Φ given in the table. As a result, this will yield the system of six equations.

In the present work, we consider a modification of the standard model on the basis of exponential representation of the coefficient C_2 and suggest a condition which determines the expediency of using this modification.

Initial System of Equations and Numerical Procedure for Their Solution. The system of Reynolds equations which is closed by the two-parameter model of turbulence is often written in the following form [1]:

$$\frac{\partial\Phi}{\partial t}+u\frac{\partial\Phi}{\partial x}+\frac{1}{r}v\frac{\partial(r\Phi)}{\partial r}-\frac{\partial}{\partial x}\left(\Gamma_\Phi\frac{\partial\Phi}{\partial x}\right)-\frac{1}{r}\frac{\partial}{\partial r}\left(\Gamma_\Phi\, r\frac{\partial\Phi}{\partial r}\right)=S_\Phi\,. \qquad (1)$$

The interpretation of the quantities Φ, S_Φ, and Γ_Φ for each equation of transfer is given in Table 1. The SIMPLE procedure was used for numerical solution of the system of equations.

The above-given system of equations and numerical procedure were used for modeling a combined flow with a twist studied in [8] experimentally. The setup consisted of two swirlers which formed coaxial rotating flows. The axial and tangential components of the velocity were measured and calculated in the zone of mixing of the two flows. Experimentally measured profiles at the inlet to the mixing zone were used as the inlet boundary conditions. In Fig. 1a, the curves show calculation of the axial velocity components by the standard k–ε model.

Results of this calculation do not predict any recirculation zone, although the measurements show its presence between the cross sections $x = 1$ and $x = 4$. Agreement between the experimental and calculated data is unsatisfactory in this zone, which indicates the pitfalls in turbulence modeling. The k–ε model predicts

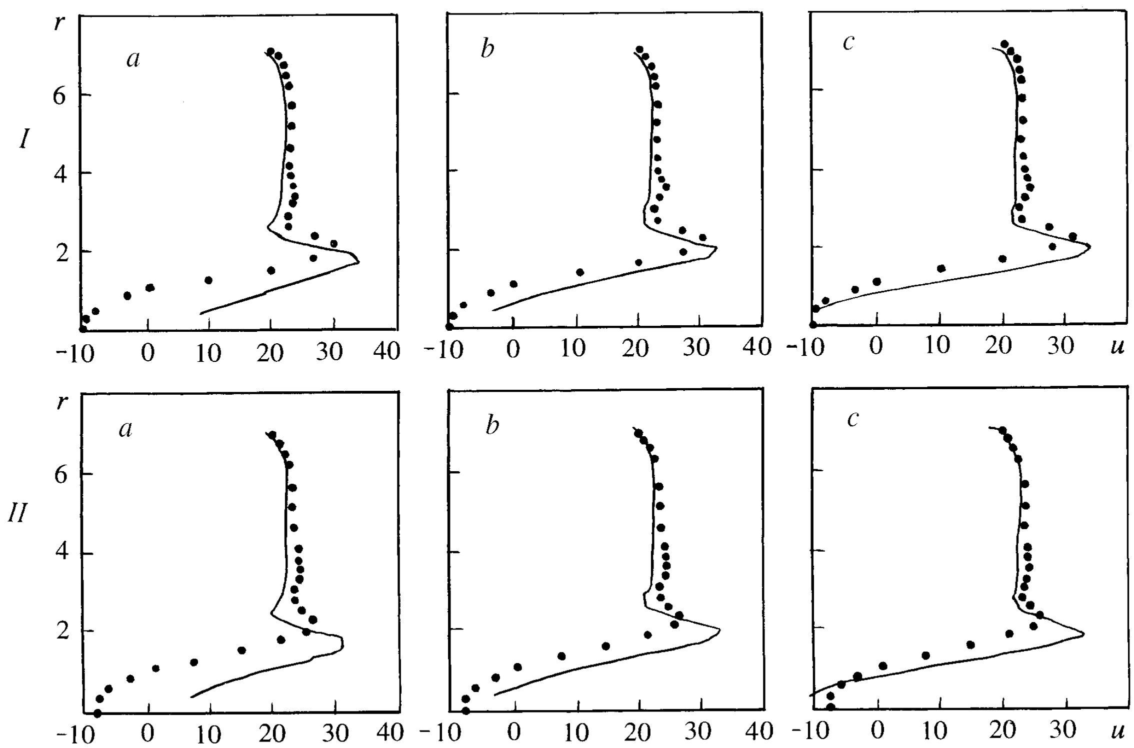

Fig. 1. Profiles of the axial component of the velocity for two measurement cross sections located in the central recirculation zone [dots, experiment [8]; curves, calculation according to the standard model (a), according to modifications (3) (b) and (5) (c)]: I, x/D = 0.8; II, 1.1. r, cm; u, m/sec.

insufficient diffusion near the zone of recirculation. The rotation of the flow causes a flow curvature in the entire region of the flow, which in turn is the reason for the considerable changes in the Reynolds stresses and, consequently, for the increase in the rate of turbulent diffusion.

Modified *k*–ε Model. The modeling of turbulence used in the present work is based on the Bradshaw assumption [9] in which the effect of flow curvature is allowed for as follows:

$$l = l_0 \,(1 - \beta\, \mathrm{Ri})\,, \tag{2}$$

where Ri is the Richardson number, which is a measure of additional turbulent stresses caused by the flow curvature.

Launder [7] suggested a simple modification of the constant C_2 in the k–ε model in the form

$$C_2 = 1.92 - C_{2\mathrm{w}}\, \mathrm{Ri}_{\mathrm{w}}\,, \tag{3}$$

where

$$\mathrm{Ri}_{\mathrm{w}} = \frac{\dfrac{w}{r^2}\dfrac{\partial}{\partial r}(rw)}{\left(\dfrac{\partial u}{\partial r}\right)^2 + \left(r\dfrac{\partial}{\partial r}\dfrac{w}{r}\right)^2}\,, \tag{4}$$

and $C_{\mathrm{w}2}$ is the empirical coefficient at the zeroth value of which the model coincides with a standard one.

We carried out a series of calculations using the above modification at several different values of the coefficients C_{w2}. Figure 1b presents results of the calculation for C_{w2} = 1.62. This value of the coefficient c_{w2} gives the best agreement with experimental data.

In Fig. 1b, we compared the experimental and calculated values of the axial velocity in different cross sections. It should be noted that modification (3) predicts the existence of the recirculation zone. However, although it gives satisfactory agreement between the calculated and experimental data, it is still desirable to have a better coincidence of the calculated data near the central line.

It has been shown by the series of experiments that an increase in the coefficient C_{w2} successively improves the agreement between calculations and experiments. But an increase above 1.62 in C_{w2} leads to a gradual decrease in the recirculation zone. This tendency is accompanied by the appearance of the negative values of the coefficient C_2, which is unrealistic.

With the purpose of further development of the model, in the present work we employed the following modification of the coefficient C_2:

$$C_2 = 1.92 \exp (2\alpha_w \mathrm{Ri}_w + 2\alpha_c \mathrm{Ri}_c) . \tag{5}$$

In this expression, α_c and α_w are the empirical constants, Ri_w is the rotational Richardson number, and Ri_c is the Richardson curvature number obtained by the Militzer method [10]:

$$\mathrm{Ri}_c = \frac{\sqrt{u^2 + v^2}}{R_c \left[\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial v}{\partial x} \right]} , \tag{6}$$

where the radius of curvature of the flow R_c is calculated from the formula

$$R_c = \frac{(u^2 + v^2)^{3/2}}{uv \left[\frac{1}{r} \frac{\partial}{\partial r} (ru) - \frac{\partial u}{\partial x} \right]} . \tag{7}$$

The exponential form of modification is selected in order that the negative values of C_2 be eliminated. Several different values of the coefficients α_c and α_w were tested in the calculations. The best results were obtained for $\alpha_w = -0.75$ and $\alpha_c = -2.0$. Figure 1c shows a comparison of the axial profile of velocity. At the beginning of the mixing zone (the recirculation zone included), the modified k–ε model with the above values of the coefficients gives a better correlation than the standard k–ε model. However, in the downstream zone, the modified model provides a less satisfactory agreement with experimental data than the standard model. A similar tendency is typical of the profile of the tangential velocity component. The modified k–ε model specifies the peaks of the tangential velocity which are not obtained in calculation according to the standard model and gives better results in the recirculation zone. Moreover, the axial velocity is restored much slower than in the calculations according to the standard model and is in better agreement with experimental data.

The results obtained according to the suggested k–ε model demonstrate that the two-equation turbulence model can be modified for determining more accurate fields of velocities, especially in the recirculation zone. However, as follows from the above-said, in the case of use of the modification for the entire region the agreement between the experimental and calculated data can become poorer in the zones with a slight flow curvature.

In what follows, we made an attempt to improve convergence for the entire studied region by further modification of the computational model. At the first stage we considered flow with recirculation without a

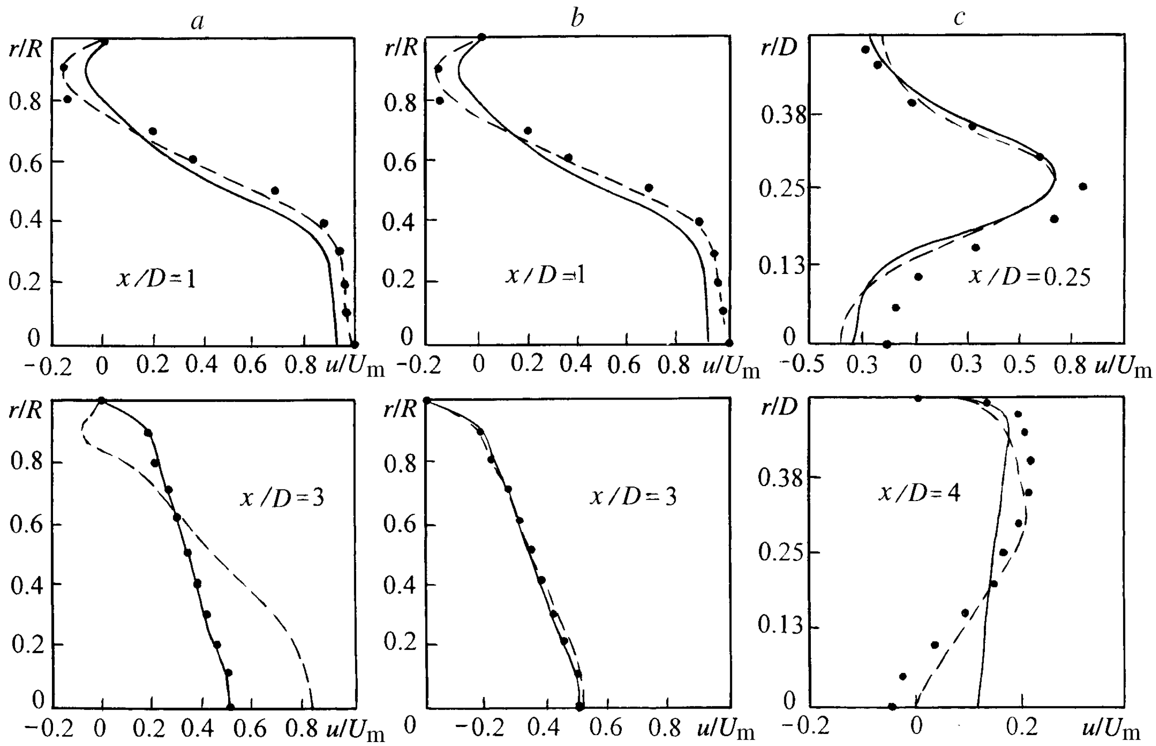

Fig. 2. Profiles of the axial component of the velocity for two measurement cross sections [dots, experiment [11] (a and b), [12] (c); dashed curves, calculation according to modifications (5) (a), (8) (b), and (10) (c); solid curves, calculation according to the standard model].

twist. For comparison we used the data of the experimental study [11] of flow in an abruptly expanding (diverging) tube with a ratio of large and small diameters equal to two. Figure 2a presents results of the calculation in comparison to experimental data. An analysis shows that the modified model (5) gives a better agreement in the recirculation zone ($x/D = 1$), where relatively higher values of the Ri_c number occur. However, in the downstream region ($x/D = 3$), the predicting ability of the modified model becomes much worse than that of the standard model. This is explained by the fact that the eddy viscosity calculated by (5) is lower, especially downstream, than that calculated using the standard model. This leads to a decrease in transverse turbulent diffusion and incorrect prediction of all the quantities entering into the system of equations (1). With the aim of developing the technique of calculation in noncirculating regions the following approach was suggested. The coefficient C_2 is corrected using expression (5) only in the regions with large Ri_c numbers. We made a number of attempts for better agreement of numerical and experimental data, which resulted in the following expressions determining when a correction of C_2 is necessary and when not:

$$C_2 = 1.92 \exp (2\alpha_c \mathrm{Ri}_c) \quad \text{when} \quad \mathrm{Ri}_c \geq 0.2 \,, \qquad C_2 = 1.92 \quad \text{when} \quad \mathrm{Ri}_c \leq 0.2 \,. \tag{8}$$

Figure 2b presents the results obtained using this modification. We can see that the modification suggested has led to an improvement of the agreement within the entire region studied with an insufficient increase in the consumption of computer time.

Successful application of the hybrid approach to a recirculation flow was tested for a recirculation flow with a twist.

Preliminary numerical calculations showed that in the case of application of modification (5) to rotating flows computational difficulties arise; these difficulties are caused by the fact that the rotational number

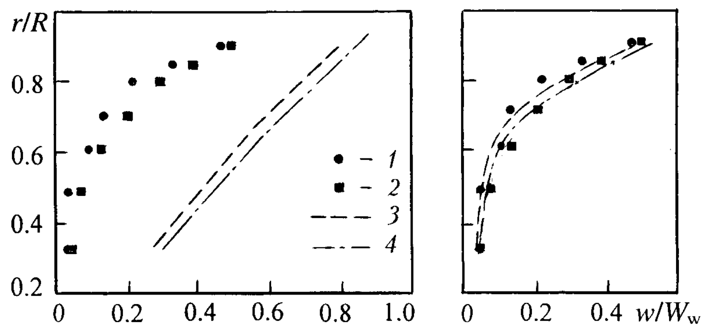

Fig. 3. Calculation according to the standard k–ε model: 1, experimental data for x/D = 40; 2, the same, 70; 3, calculation for x/D = 40; 4, the same, 70.

Ri_w takes on nearly zero values at the points near the axis of symmetry, which makes it difficult to provide the stability of solution. To improve convergence, we used here the gradient number Ri_g instead of the rotational number Ri_w:

$$\mathrm{Ri}_g = \frac{2\dfrac{w}{r^2}\dfrac{\partial}{\partial r}(rw)}{\left(\dfrac{\partial u}{\partial r}\right)^2 + \left(\dfrac{\partial w}{\partial r}\right)^2} . \tag{9}$$

As a test problem, we took the experimental study made by Dellenback [12], who studied a rotating flow of water in a channel with an abrupt divergence. Rotational motion was generated by a tangential slot inlet located upstream. The velocity was measured by a Doppler anemometer. The diameters of the input and output pipes were 50.6 and 98.6, respectively, with a length of the experimental chamber of 1041.1 mm.

We made a number of computational attempts at different rotational numbers in order to select conditions of best agreement between experimental and calculated data. As a result we suggested the following modification:

$$C_2 = 1.92 \exp\left(4\mathrm{Ri}_c + \frac{S/3 - 1.4}{\mathrm{Ri}_g + 1}\right) \text{ when } \mathrm{Ri}_c \geq 0.2 , \quad C_2 = 1.92 \text{ when } \mathrm{Ri}_2 < 0.2 , \tag{10}$$

$$S = \frac{\int_0^R u r^2 w dr}{r \int_0^R u r^2 dr} .$$

Figure 2c presents results of the calculation according to modification (10) and experimental data for the axial profile. As is seen from the graphs, both the standard and hybrid models give similar results for a flow in the initial part of the channel. Further downstream, agreement between the calculated data obtained using modification (10) and the experimental data is much better than in the case of use of the standard model. In [13], the algebraic model for Reynolds stresses is used. Despite the fact that it refers to a higher level of turbulence modeling, we did not obtain a noticeable improvement of prediction of the flow. But in this case the calculation time increased severalfold.

The approach described above was employed to calculate the tangential component of the velocity in a rotating tube, experimental data for which are given in [14]. A 32-mm-diameter tube with a hydraulically smooth surface was used in the experiment. Water was supplied from the tank to a rectilinear portion with a length of 60 gauges; then it entered a rotating tube with a 160-gauge length. The axial and tangential velocities were measured in several cross sections from the beginning of the rotating tube. The calculation results are given in Fig. 3. Their agreement for the modified model is much better than for the standard model and is acceptable for practice.

Conclusions. Two-parameter models can successfully be modified for calculation of flows with a considerable flow curvature. In this case, modification does not lead to a substantial complication of the computational algorithm and an increase in the consumption of computer time. It is worthwhile to modify the standard model in those subregions where the flow curvature is considerable. The Richardson number can successfully be used for determining these subregions.

NOTATION

t, time; x and r, axial and radial coordinates; u, v, and w, axial, radial, and tangential components of the velocity; U_m, mean flow rate; W_w, linear rotational velocity of the tube surface; p, pressure; k, kinetic energy of turbulence; ε, rate of dissipation of the turbulent kinetic energy; ν, molecular viscosity; ν_τ, eddy viscosity; ν_{eff}, effective viscosity; Ri, Richardson number; y, distance from the wall; l_0, mixing length for a simple flow; L, mixing length; C_1, C_2, C_{w2}, C_μ, σ_k, and σ_ε, coefficients of the k–ε model; β, empirical coefficient; R, tube radius; D, tube diameter; Φ, sought parameter; S_Φ, source term; Γ_Φ, coefficient of diffusion; S, degree of rotation.

REFERENCES

1. Yu. V. Lapin and M. Kh. Strelets, *Internal Flows of Gas Mixtures* [in Russian], Moscow (1989).
2. M. Nallasamy, *Comput. Fluids*, **3**, 151–194 (1987).
3. J. H. Ferziger, *J. Comput. Phys*., **69**, 1–48 (1973).
4. M. Nikjooy and Y. C. Mongia, *J. Heat Fluid Flow*, **12**, 12–19 (1991).
5. S. Fu, P. G. Huang, B. E. Launder, and M. A. Leschziner, *J. Fluid Eng*., **110**, 216–221 (1998).
6. W. P. Jones and A. Pascau, *J. Fluid Eng*., **111**, 248–252 (1989).
7. B. E. Launder, *Turbulence Models and Their Experimental Verifications, Two-Equation Turbulence Models*, Imperial College Rep. (1973).
8. B. T. Vu and C. F. Gouldin, *AIAA Paper* No. 80.0076 *presented at the AIAA Aerospace Sci. Meeting* (1980).
9. P. Bradshaw, *J. Fluid Mech*., **36**, 1007–1048 (1971).
10. J. Militzer, *Dual Plane Parallel Turbulent Jets: The Measurement and Prediction of Mean Velocity Field*, Ph.D. Thesis, Waterloo (1977).
11. J. E. Drewry, *AIAA J*., **16**, 313–317 (1987).
12. P. A. Dellenback, *Heat Transfer and Velocity Measurements in Turbulent Swirling Flow through an Abrupt Axisymmetric Expansion*, Ph.D. Thesis, Tempe, Arizona (1986).
13. B. K. Sultanian, *Numerical Modeling of Turbulent Swirling Flow downstream of an Abrupt Pipe Expansion*, Ph.D. Thesis, Tempe, Arizona (1984).
14. M. Murakami and K. Kiluyama, *Trans. ASME, J. Fluid Eng*., **102**, 97–103 (1980).